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RM-84-14-ONR

**INFORMATION-DECAY PURSUIT OF DYNAMIC
PARAMETERS IN STUDENT MODELS**

Robert J. Mislevy

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This research was sponsored in part by the
Cognitive Science Program
Cognitive and Neural Sciences Division
Office of Naval Research, under
Contract No. N00014-88-K-0304
R&T 4421552

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Educational Testing Service
Princeton, NJ

April 1994

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REPORT DOCUMENTATION PAGEForm Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE April 1994	3. REPORT TYPE AND DATES COVERED Final
4. TITLE AND SUBTITLE Information-Decay Pursuit of Dynamic Parameters in Student Models			5. FUNDING NUMBERS G. N00014-88-K-0304 PE. 61153N PR. RR 04204 TA. RR 04204-01 WU. R&T 4421552
6. AUTHOR(S) Robert J. Mislevy			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Educational Testing Service Rosedale Road Educational Testing Service			8. PERFORMING ORGANIZATION REPORT NUMBER RM-94-14-ONR
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Cognitive Sciences Code 1142CS Office of Naval Research Arlington, VA 22217-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER N/A
11. SUPPLEMENTARY NOTES None			
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified/Unlimited			12b. DISTRIBUTION CODE N/A
13. ABSTRACT (Maximum 200 words) Learning processes are often analyzed using a model with a structure that is assumed to apply to all points in time, but with one or more parameters that may change over time. In some applications, such as learning theory, the goal is to model the character of change itself; observations at all time points are used to estimate, for example, learning curves. In other applications, such as tutoring systems, the goal is to make decisions based on status at each point in time. Current observations depend directly on current value of the parameter(s) of interest but recent observations were produced by values that were probably similar if not identical. This note describes and illustrates approximations for the current value of such a parameter, using models that posit no change but downweight the influence of observations as they recede in time. The idea is similar to that of robust estimation procedures that downweight the influence of outliers. The applicability of these approaches to intelligent tutoring systems (ITS's) that use Bayesian inference networks to update student models is discussed.			
14. SUBJECT TERMS Bayesian inference networks, causal probability networks, intelligent tutoring systems, robust estimation			15. NUMBER OF PAGES 32
			16. PRICE CODE N/A
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT SAR

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March, 1994

Accession For	
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DTIC	TAB <input type="checkbox"/>
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Justification	
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This work was supported by Contract No. N00014-91-J-4101, R&T 4421573-01, from the Cognitive Science Program, Cognitive and Neural Sciences Division, Office of Naval Research, and by Armstrong Laboratories of the United States Air Force under Prime Contract No. F33615-90-D-0008 to the University of Pittsburgh. I am grateful to Drew Gitomer and Kathy Sheehan for helpful discussions and comments.

Information-Decay Pursuit of Dynamic Parameters in Student Models

Abstract

Learning processes are often analyzed using a model with a structure that is assumed to apply to all points in time, but with one or more parameters that may change over time. In some applications, such as learning theory, the goal is to model the character of change itself; observations at all time points are used to estimate, for example, learning curves. In other applications, such as tutoring systems, the goal is to make decisions based on status at each point in time. Current observations depend directly on current value of the parameter(s) of interest, but recent observations were produced by values that were probably similar if not identical. This note describes and illustrates approximations for the current value of such a parameter, using models that posit no change but downweight the influence of observations as they recede in time. The idea is similar to that of robust estimation procedures that downweight the influence of outliers. The applicability of these approaches to intelligent tutoring systems (ITS's) that use Bayesian inference networks to update student models is discussed.

Key words: Bayesian inference networks, causal probability networks, intelligent tutoring systems, robust estimation.

Introduction

This note concerns inference in situations in which the *form* of a stochastic model is assumed constant, but values of certain *parameters* may change over time. Assuming we could carry out likelihood-based or Bayesian inference if there were no secular change, we discuss approximations that retain these forms but discount information from older observations. Two "decaying information" approximations are weighted likelihoods and successively adjusted posterior distributions. The resulting estimators tend to lag behind a monotonically changing parameter, in what might be called a "pursuit" strategy. The rationale parallels that of reducing the influence of outliers in robust estimation (Huber, 1981): Decaying-information estimators are less efficient than full-information estimators if the parameters are in fact constant, but they provide less biased approximations of current values when trends do exist. Using a robust version of a simpler model, as opposed to fitting more a complex model, provides serviceable, if not optimal, approximations across a variety of departures from the basic form.

The motivating problem arose in the context of an intelligent tutoring system (ITS) for trouble-shooting aircraft hydraulics (Gitomer, Steinberg, & Mislevy, in press). In HYDRIVE, students' actions are modeled as probabilistic functions of parameters that characterize aspects of a student's system understanding, strategic knowledge, and procedural skills. The values of these parameters increase with experience and practice, but with different patterns and rates for different students. Certain instructional decisions would be based on current values of these parameters at a given point in time, but these values are not known with certainty; only imperfect information in the form of a history of actions is available. We presume that an action's evidential value about student-model parameters declines over time. Our purpose here is to begin exploring the potential of information-decay approximations in this context; the second running example is a simplified version of the problem. Formal investigations of the properties of the methods are beyond the scope of the present paper.

Running Examples

Example 1: Animal Learning

Suppose Session t in an learning experiment consists of M_t trials in which the animal chooses either the correct or the incorrect path in a maze. It may be reasonable to

model these trials as independent Bernoulli variables with a common probability θ_t of a correct choice. As in the simple conditioning model discussed below, θ may change from one session to the next. Choices in Session t , say $x_t = (x_{t1}, \dots, x_{tm_t})$, thus provide direct evidence about θ_t , but indirect and successively weaker evidence about values of θ_{t+1} , θ_{t+2} , and so on.

Example 2: Educational Testing

An examinee's responses to equally-difficult test items are observed in sequence. The variable of interest is her unobservable "proficiency," say θ , the probability of answering items correctly. Item responses are modeled as conditionally independent given θ . (More complex models allow for nonexchangeable items, multidimensional θ , and multiple, perhaps unordered or partially-ordered, response categories.) Typically θ is assumed to be constant over the period of observation. Secular trends may exist, however, as when students overcome initial nervousness or learn during the course of testing. In a context similar to that of HYDRIVE, Kimball (1982) models the probability that a student will apply a given calculus rule in appropriate circumstances as constant within an exercise, but updates the estimate between problems.

Setup and Notation

Let t index time points, and consider observations of M_t iid variables $x_t = (x_{t1}, \dots, x_{tm_t})$ at Time t , conditional on the parameter θ_t . Let $p(x_{ij}; \theta_t)$ denote their common pdf. The outcomes observed at Time t induce a likelihood function for θ_t :

$$L(\theta_t | x_t) = \prod_{j=1}^{M_t} p(x_{tj}; \theta_t). \quad (1)$$

Denote by $X_t = (x_1, \dots, x_t)$ the sequence of observations up through Time t ; that is, x_1 represents the possibly vector-valued observation from Time 1, governed by θ_1 , x_2 the observation from Time 2, governed by θ_2 , and so on. Let $p_{t-1}(\theta_t | X_{t-1})$ represent belief about θ_t at Time $t-1$, or prior to observing x_t , starting with $p_0(\theta_1)$. Belief about θ_t posterior to observing x_t is obtained by Bayes Theorem as

$$p_t(\theta_t | X_t) \propto p(x_t | \theta_t) p_{t-1}(\theta_t | X_{t-1}). \quad (2)$$

In the context of Kimball's calculus ITS, θ_t would represent the student-model parameter value for Exercise t , and x_{tj} , for $j=1, \dots, M_t$, would represent instances of applying or not applying the target rule when it applies during the course of the exercise. $p_{t-1}(\theta_t | X_{t-1})$ would represent the belief about θ_t at the beginning of the problem, based on actions in previous problems and a reinforcement function; $p(x_t | \theta_t)$ would be a binomial likelihood based on x_t ; and $p_t(\theta_t | X_t)$ would be the posterior for θ_t combining both sources of information.

If θ were constant over time—that is, $\theta_t \equiv \theta$ for all t —the posterior shown as (2) after observations at time points 1 through T would simply be proportional to an initial prior for θ and a joint likelihood function $L(\theta | X_T)$ combining observations from a time points in the same way:

$$\begin{aligned} p_T(\theta | X_T) &\propto p(x_T | \theta) p(\theta | X_{T-1}) \\ &\propto p(x_T | \theta) p(X_{T-1} | \theta) p_0(\theta) \\ &= \prod_{t=1}^T L(\theta | x_t) p_0(\theta) \\ &= L(\theta | X_T) p_0(\theta). \end{aligned} \tag{3}$$

Bayesian inference over time would be based on the sequence of posterior distributions

$$p_t(\theta | X_t) \propto p_{t-1}(\theta | X_{t-1}) L(\theta | x_t), \tag{4}$$

each built up from $p_0(\theta)$ by multiplying another likelihood term $L(\theta | x_t)$ as given in (1), but all with the same θ .

Suppose, however, that the value of θ varies over time, perhaps as a stochastic function $p(\theta_t | X_{t-1}, \gamma)$ of X_{t-1} , and/or an unknown parameter γ . An example is Estes's (1950) model for the probability of the keyed response on Trial t under simple conditioning, with initial probability θ_0 and learning parameter γ :

$$\theta_t = 1 - (1 - \theta_0)^{-\gamma}. \tag{5}$$

Bayesian inference about θ_T after observing X_T requires expanding the posterior given in (2) as follows:

$$\begin{aligned}
 p_T(\theta_T | X_T) &\propto p(x_T | \theta_T) p_{T-1}(\theta_T | X_{T-1}) \\
 &= p(x_T | \theta_T) \int p_{T-1}(\theta_T | X_{T-1}, \gamma) p(\gamma | X_{T-1}) d\gamma.
 \end{aligned} \tag{6}$$

In contrast to (2), the more complex form of (4) requires specifying and manipulating functions that model the nature of change, $p(\theta_T | X_{T-1}, \gamma)$ and $p(\gamma | X_{T-1})$, which harbor the possibility of specification error.

We aim to approximate (6) without modeling the nature of change, for situations in which the primary interest is status at Time T rather than the nature of change, which may vary across cases. This approach is analogous to that of robust procedures that limit the influence of aberrant observations in time series processes (e.g., Hampel et al., 1986, Section 8.3; Martin, 1979). The two approximations sketched below replace the term $p(\theta_T | X_{T-1})$ in (6) with a simpler version based on the assumption of stasis over time, as in (3) and (4). The first approximation uses a weighted likelihood, such that the influence of observations decays over time. The second replaces each posterior in the sequence with an attenuated version, shrinking each time toward an initial, less informative, prior.

Decaying Likelihoods

Suppose we model all observations as if they were iid with a common parameter $\theta \equiv \theta_T$. This approximation is correct insofar as x_T itself is concerned, but less trustworthy for observations further back in the sequence. Our decreasing confidence in this expedient can be reflected by a weighted likelihood:

$$\begin{aligned}
 L^*(\theta | X_T) &= \prod_{i=1}^T [p_T(x_i; \theta)]^{w_i} \\
 &= \prod_{i=1}^T \left[\prod_{j=1}^{M_i} p(x_{ij}; \theta) \right]^{w_i},
 \end{aligned} \tag{7}$$

where $0 < w_1 \leq \dots \leq w_T \leq 1$. This leads to the corresponding weighted posterior $p_T^*(\theta | X_T) \propto L^*(\theta | X_T) p_0(\theta)$.

As with modeling the form of change, choosing a weighting scheme involves choices; in particular, how quickly to discount information as it ages. The choice matters less for observations more distant in time, however—exactly where modeling the true

relationship between current and past values of θ is apt to be most difficult. One option is a geometric rate of decay, with parameter δ : that is,

$$w_t = \delta^{T-t} \quad (8)$$

in (7). Table 1 shows the sequences of decreasing weights that correspond to $\delta=1$, .95, .75, and .5; that is, ranging from no decay down to a half life of one unit. Example 1 below illustrates its use. An alternative to a smooth rate of decay is to posit weights of 1 for recent time points, and weights of zero for all points further than, say, c units in the past. This scheme is used in Example 2.

Point estimates of θ based on (7) are analogous to robust M-estimators (Huber, 1981), except that observations' weights are based on recency rather than typicality. For cases in which θ is in fact constant over time, (7) is an example of what Arnold and Strauss (1988) call a "pseudo-likelihood;" under regularity conditions, its maximum is a consistent, asymptotically normal, estimator of θ . Likelihood point estimates of θ based on (7) tend to the correct central tendency, but less efficiently than standard full-information maximum likelihood estimators.

[Table 1]

Example 1, continued: Estes's Model for Simple Conditioning

Table 2 presents data generated according to the Estes simple conditioning model (5), and weighted-likelihood approximations of θ obtained with various geometric rates of decay. The second column gives the values of θ_t produced recursively from model (5) with $\theta_0=.2$ and $\gamma=.1$. The sequence of Bernoulli variables x_t in the next column was generated, each with the corresponding θ_t as the probability of observing a '1' ($M_t=1$ for all t). If we were to assume θ were constant over time and impose a uniform prior, the posterior for θ at Time T would be the beta distribution

$$\begin{aligned} p_T(\theta | \mathbf{x}_T) &\propto \prod_{t=1}^T \theta^{x_t} (1-\theta)^{1-x_t} \\ &= \theta^{s_T} (1-\theta)^{T-s_T}, \end{aligned}$$

with $s_T = \sum_{i=1}^T x_i$, the number of successes. The posterior mean of θ at Time T would be the average of all observations up through that point, $\hat{\theta}_T = S_T/T$. A geometric rate of likelihood decay approximation with decay parameter δ replaces T and s_T with

$$T^* = \sum_{i=1}^T \delta^{T-i} \quad \text{and} \quad s_T^* = \sum_{i=1}^T \delta^{T-i} x_i.$$

[Table 2]

Table 2 gives decaying likelihood posterior means, namely weighted means $\hat{\theta}_T^* = s_T^*/T^*$, with $\delta=1$ (no decay), .95, .75, and .50. While all of these sequences of weighted means tend to lag behind the generating probabilities, faster rates of decay decrease the drag; they pursue the true values more closely *on the average*. But faster rates of decay also increase the variance of the estimate. As in robust estimation, less information is used in the hope that the information foregone is most apt to be misleading. In this example, means with $\delta=.5$ are less biased than those obtained with $\delta=.75$, but the more severe trimming causes the $\delta=.5$ means to have a larger mean squared error.

Example 2, continued: An Inference Network for Sequential Testing

In this example, an examinee's proficiency θ is posited to have only three possible values, -1, 0, and +1, with prior probabilities of .25, .50, and .25. There are five test items, responses to which are assumed iid for simplicity.¹ The conditional probabilities of a correct item response at the three θ values are .10, .40, and .70 respectively. Numerical results will be presented in the idiom of Bayesian inference networks.

Bayesian inference networks (also referred to as causal probability networks and influence diagrams) are systems for representing and carrying out probability-based inference in networks of interrelated variables (see Andreassen, Jensen, & Olesen, 1990; Lauritzen & Spiegelhalter, 1988; Pearl, 1988, and Shafer & Shenoy, 1988). The starting point is a directed acyclic graph (DAG) representation of the conditional independence relationships among the variables; Figure 1 shows the DAG for this example. A

¹ See Wainer et al., 1990, on adaptive sequential testing with a continuous real-valued proficiency parameter θ and items parameterized individually in terms of their regressions on θ .

representation of the joint probability distribution is constructed in terms of products of the distributions of subsets of interrelated variables (cliques) divided by distributions of intersecting subsets. A corresponding "join tree" representation with the property of single-connectedness makes it possible to propagate coherently the consequences of new information throughout the network, employing only calculations local to cliques and their neighbors. Figure 2 is the join tree for the current example. Calculations are carried out by means of operations on "potential tables," which comprise values proportional to the joint probabilities of the potential value combinations of all the variables in a clique (see Mislevy, *in press(a)*, for a simple worked-through example). Commercially available computer programs for structuring and using Bayesian inference include ERGO (Noetic Systems, Inc., 1991) and HUGIN (Andersen, Jensen, Olesen, & Jensen, 1989).

[Figures 1 & 2]

Table 3 is a trace of the updating of potential tables as the response sequence (0,0,1,1,1) is absorbed one item at a time. The matrices for the cliques and the clique intersection correspond to the depiction in Figure 2. Note how the marginal table for θ shifts belief toward -1 after the first two incorrect responses, then increases gradually upward as the next three correct responses arrive. Table 4 shows updating with information-decay weights of 1 for the current and previous two observations, and 0 for observations further back in the sequence. These values were achieved in ERGO by, at each time point, absorbing information from x_t and, once $t > 3$, retracting information corresponding to x_{t-3} and any earlier observations. Table 5 gives the means of the successive θ posteriors, starting with the prior. (The "Geometric decay" and "Attenuated posterior" rows are discussed below.) The full-information and trimmed-likelihood posterior means are identical for the first three responses by construction, but they depart for $t=4$ and $t=5$ when the leading responses are trimmed from the sequence.

[Tables 3-5]

We now implement the geometrically decaying likelihood scheme, with $\delta = .75$. This is effected in the inference network by, at each time point, entering the current finding with the correct conditional odds ratios for right and wrong responses, and for past time points, re-entering the findings with odds ratios multiplied by δ^{T-t} . The conditional

probabilities that effect the decaying likelihood are shown as Table 6.² Findings are entered as if anew at each time point, with potential tables reflecting the decayed odds updated by the corresponding observed responses. Table 7 traces the results of this procedure. As seen in Table 5, the results after the final string of correct responses lead to higher posterior means for θ than the straight likelihood solution, but not as high as those of the "three most recent items" scheme. The influence of the initial incorrect responses remains, albeit with declining influence.

[Tables 6 & 7]

Attenuated Posteriors

A disadvantage of the decaying likelihood approach discussed above is having to retain and re-analyze, at each time point, some or all of the data from past time points. An alternative is to modify the posterior obtained at each point to achieve the same effect of successively discounting the influence of earlier observations. The form of the "attenuated posterior" approximation is thus defined recursively as

$$p_T^*(\theta|X_T) \propto p(x_T|\theta_T)g[p_{T-1}^*(\theta|X_{T-1})], \quad (9)$$

where $g[\cdot]$ is a function that "flattens" the previous posterior so that the current observations will have a relatively greater influence. For example:

1. When employing conjugate priors, one can modify the parameters of each successive distribution so as to remain within the same parametric family. Suppose x follows a Poisson distribution with parameter θ constant over time, and belief about θ at Time $T-1$ is expressed as $\text{Gamma}(\alpha_{T-1}, n_{T-1})$. The usual posterior after observing x_T is $\text{Gamma}(\alpha_{T-1}+x_T, n_{T-1}+1)$. One possible attenuated conjugate prior for Time T would be $\text{Gamma}(\delta\alpha_{T-1}, \delta n_{T-1})$, where $0 < \delta \leq 1$, a distribution with the same functional form and mean but more dispersed. The resulting posterior would be $\text{Gamma}(\delta\alpha_{T-1}+x_T, \delta n_{T-1}+1)$.

² The probabilities for $x=1$ and $x=0$ given $\theta=-1$ are .1 and .9, for example, for an odds ratio of .111; the odds ratio for the same conditional probabilities with $\text{lag}=1$ is therefore $.111 \cdot .75 = .192$, implying probabilities of .161 and .839 respectively. For $\text{lag}=2$, the odds ratio becomes $.111 \cdot .75 \cdot .75 = .291$, implying conditional probabilities of .225 and .775. (Recall that an odds ratio of 1 is tantamount to "no evidence.")

2. Suppose θ takes one of the values $(\theta_1, \dots, \theta_K)$, and the observation of x induces the likelihoods $(L_1(x), \dots, L_K(x))$. Let belief at Time $T-1$ be represented by the vector of probabilities $(\pi_{1,T-1}, \dots, \pi_{K,T-1})$, where $\sum_k \pi_{k,T-1} = 1$. By Bayes Theorem, the posterior for θ after the observation of x_T is obtained as

$$(\pi_{1,T}, \dots, \pi_{K,T}) = (CL_1(x_T)\pi_{1,T-1}, \dots, CL_K(x_T)\pi_{K,T-1}),$$

where $C = 1 / \sum_k L_k(x_T)\pi_{k,T-1}$. Let $(\pi_{10}, \dots, \pi_{K0})$ represent a "baseline" (usually flatter) prior distribution for θ , toward which we wish to bias belief at each stage. We can define a decayed-information version of the posterior by using for the prior a mixture of the $T-1$ posterior and the baseline prior:

$$(\pi_{1,T}^*, \dots, \pi_{K,T}^*) = (C^* L_1(x_T)\pi_{1,T-1}^*, \dots, C^* L_K(x_T)\pi_{K,T-1}^*),$$

where

$$\pi_{k,T-1}^* = \delta \pi_{k,T-1} + (1 - \delta) \pi_{k0} \quad (10)$$

and $C^* = 1 / \sum_k L_k(x_T)\pi_{k,T-1}^*$.

Example 2, continued

This example uses the same inference network and realized response pattern discussed above. After each item, an attenuated posterior replaces the true Bayesian posterior, which is mixed with a proportion of .75 with the baseline prior (.25, .50, .25). Table 8 traces the successive updating of potential tables. For each time point, the "After Item t " tables represent updating from the previous set of beliefs; "Before Item $t+1$ " represent the results of mixing the resulting posterior with the baseline prior as in (10) and propagating this change through the network. The trace of the posterior mean for θ , appearing as the final row of Table 5, is similar to that of the geometrically decaying likelihood.

[Table 8]

Discussion

This paper begins to explore ways to account for learning that might be useful intelligent tutoring systems (ITSs). In the HYDRIVE ITS (Gitomer, Steinberg, & Mislevy, in press), students work through a series of adaptively selected problems. Problem selection and interventions within problems are guided by current estimates in the student model. A Bayesian inference network is employed to update the student model within a problem, under the assumption that its parameters are stable during the course of observation. (An exception is upward adjustment following episodes of direct instruction.) By the end of a problem, enough information has been acquired to make reasonably well informed inferences about the student status, at least for parameters dealing with aspects of knowledge that the problem probes. At the beginning of each problem, though, performances on previous problems presumably convey some information about the student's status. We wish to exploit this information, without either assuming that the student model does not change over the course of problems or attempting to model the likely nature of change from one problem to the next (as is done, for example, Anderson's LISP tutor (Anderson & Reiser, 1985) and Kimball's (1982) calculus tutor, which use variants of learning models such as Estes's).

While both the decaying-likelihood and attenuated-posterior approaches discussed above achieve the desired effect, the decaying-likelihood approach requires raw data to be maintained for subsequent partial retraction. The attenuated-posterior approach works with temporally local quantities only. This seems preferable in a context such as ours, where it is advantageous to minimize bookkeeping, computation, and data maintenance. An additional advantage is that this approach is compatible with the storage- and calculation-reducing scheme described in Mislevy (in press(b)) for reducing the size of Bayesian inference networks with multiple copies of iid observable variables.

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Table 1
Weights for Geometric Rate of Information Decay

t	$\delta=1$	$\delta=.95$	$\delta=.75$	$\delta=.50$
1	1.00	1.00	1.00	1.00
2	1.00	.95	.75	.50
3	1.00	.90	.56	.25
4	1.00	.86	.42	.13
5	1.00	.81	.32	.06
6	1.00	.77	.24	.03
7	1.00	.74	.18	.02
8	1.00	.70	.13	.01
9	1.00	.66	.10	.00
10	1.00	.63	.08	.00
11	1.00	.60	.06	.00
12	1.00	.57	.04	.00
13	1.00	.54	.03	.00
14	1.00	.51	.02	.00
15	1.00	.49	.02	.00
16	1.00	.46	.01	.00
17	1.00	.44	.01	.00
18	1.00	.42	.01	.00
19	1.00	.40	.01	.00
20	1.00	.38	.00	.00
21	1.00	.36	.00	.00
22	1.00	.34	.00	.00
23	1.00	.32	.00	.00
24	1.00	.31	.00	.00
25	1.00	.29	.00	.00
26	1.00	.28	.00	.00
27	1.00	.26	.00	.00
28	1.00	.25	.00	.00
29	1.00	.24	.00	.00
30	1.00	.23	.00	.00

Table 2
Estimated Time t Means with Information Decay

t	θ_t	x_t	$\delta=1$		$\delta=.95$		$\delta=.75$		$\delta=.50$	
			$\hat{\theta}_t$	error	$\hat{\theta}_t$	error	$\hat{\theta}_t$	error	$\hat{\theta}_t$	error
1	.28	0	.00	-.28	.00	-.28	.00	-.28	.00	-.28
2	.35	1	.50	.15	.51	.17	.57	.23	.67	.32
3	.41	0	.33	-.07	.33	-.07	.32	-.08	.29	-.12
4	.46	0	.25	-.21	.24	-.22	.21	-.26	.13	-.33
5	.51	1	.40	-.11	.41	-.10	.47	-.05	.58	.07
6	.56	0	.33	-.23	.33	-.23	.32	-.24	.29	-.28
7	.60	1	.43	-.17	.44	-.16	.52	-.08	.65	.04
8	.64	1	.50	-.14	.53	-.11	.65	.01	.82	.18
9	.67	0	.44	-.23	.46	-.22	.48	-.20	.41	-.26
10	.71	1	.50	-.21	.52	-.18	.62	-.09	.71	.00
11	.73	1	.55	-.19	.58	-.16	.72	-.02	.85	.12
12	.76	0	.50	-.26	.52	-.24	.53	-.23	.43	-.33
13	.78	1	.54	-.24	.57	-.22	.65	-.13	.71	-.07
14	.80	1	.57	-.23	.61	-.20	.74	-.06	.86	.05
15	.82	1	.60	-.22	.64	-.18	.81	-.02	.93	.11
16	.84	1	.63	-.21	.68	-.16	.85	.02	.96	.13
17	.85	1	.65	-.21	.70	-.15	.89	.04	.98	.13
18	.87	0	.61	-.26	.65	-.22	.67	-.20	.49	-.38
19	.88	1	.63	-.25	.67	-.21	.75	-.13	.75	-.13
20	.89	1	.65	-.24	.70	-.19	.81	-.08	.87	-.02
21	.90	0	.62	-.28	.65	-.26	.61	-.29	.44	-.47
22	.91	0	.59	-.32	.60	-.31	.46	-.45	.22	-.69
23	.92	1	.61	-.31	.63	-.29	.59	-.33	.61	-.31
24	.93	1	.63	-.30	.65	-.27	.69	-.23	.80	-.12
25	.93	1	.64	-.29	.68	-.26	.77	-.16	.90	-.03
26	.94	1	.65	-.29	.70	-.24	.83	-.11	.95	.01
27	.95	1	.67	-.28	.72	-.23	.87	-.07	.98	.03
28	.95	1	.68	-.27	.74	-.21	.90	-.05	.99	.04
29	.96	1	.69	-.27	.75	-.20	.93	-.03	.99	.04
30	.96	1	.70	-.26	.77	-.19	.95	-.01	1.00	.04
Mean				-.22		-.19		-.12		-.08
SD				.09		.09		.13		.22
MSE				.06		.04		.03		.05

Table 3

Trace of Potential Tables and Clique Intersection Table for Five Responses to IID Items:
Response Vector = (0,0,1,1,1), No Information Decay

INITIAL STATUS				AFTER ITEM 1 = 0				AFTER (0,0)			
		Item1=1	Item1=0			Item1=1	Item1=0			Item1=1	Item1=0
$\theta=-1$.025	.225	$\theta=-1$		0	.375	$\theta=-1$		0	.500
$\theta=0$.200	.300	$\theta=0$		0	.500	$\theta=0$		0	.444
$\theta=+1$.175	.075	$\theta=+1$		0	.125	$\theta=+1$		0	.056
		Item2=1	Item2=0			Item2=1	Item2=0			Item2=1	Item2=0
$\theta=-1$.025	.225	$\theta=-1$.038	.337	$\theta=-1$		0	.500
$\theta=0$.200	.300	$\theta=0$.200	.300	$\theta=0$		0	.444
$\theta=+1$.175	.075	$\theta=+1$.088	.038	$\theta=+1$		0	.056
		Item3=1	Item3=0			Item3=1	Item3=0			Item3=1	Item3=0
θ				θ				θ			
$\theta=-1$.250	$\theta=-1$.025 .225	$\theta=-1$.375	$\theta=-1$.038 .337	$\theta=-1$.500	$\theta=-1$.050 .450
$\theta=0$.500	$\theta=0$.200 .300	$\theta=0$.500	$\theta=0$.200 .300	$\theta=0$.444	$\theta=0$.178 .267
$\theta=+1$.250	$\theta=+1$.175 .075	$\theta=+1$.125	$\theta=+1$.088 .038	$\theta=+1$.056	$\theta=+1$.039 .017
		Item4=1	Item4=0			Item4=1	Item4=0			Item4=1	Item4=0
$\theta=-1$.025	.225	$\theta=-1$.038	.337	$\theta=-1$.050	.450
$\theta=0$.200	.300	$\theta=0$.200	.300	$\theta=0$.178	.267
$\theta=+1$.175	.075	$\theta=+1$.088	.038	$\theta=+1$.039	.017
		Item5=1	Item5=0			Item5=1	Item5=0			Item5=1	Item5=0
$\theta=-1$.025	.225	$\theta=-1$.038	.337	$\theta=-1$.050	.450
$\theta=0$.200	.300	$\theta=0$.200	.300	$\theta=0$.178	.267
$\theta=+1$.175	.075	$\theta=+1$.088	.038	$\theta=+1$.039	.017
AFTER (0,0,1)				AFTER (0,0,1,1)				AFTER (0,0,1,1,1)			
		Item1=1	Item1=0			Item1=1	Item1=0			Item1=1	Item1=0
$\theta=-1$		0	.188	$\theta=-1$		0	.048	$\theta=-1$		0	.010
$\theta=0$		0	.667	$\theta=0$		0	.688	$\theta=0$		0	.592
$\theta=+1$		0	.146	$\theta=+1$		0	.263	$\theta=+1$		0	.397
		Item2=1	Item2=0			Item2=1	Item2=0			Item2=1	Item2=0
$\theta=-1$		0	.188	$\theta=-1$		0	.048	$\theta=-1$		0	.010
$\theta=0$		0	.667	$\theta=0$		0	.688	$\theta=0$		0	.592
$\theta=+1$		0	.146	$\theta=+1$		0	.263	$\theta=+1$		0	.397
		Item3=1	Item3=0			Item3=1	Item3=0			Item3=1	Item3=0
θ				θ				θ			
$\theta=-1$.188	$\theta=-1$.188 0	$\theta=-1$.048	$\theta=-1$.048 0	$\theta=-1$.010	$\theta=-1$.010 0
$\theta=0$.667	$\theta=0$.667 0	$\theta=0$.688	$\theta=0$.688 0	$\theta=0$.592	$\theta=0$.592 0
$\theta=+1$.146	$\theta=+1$.146 0	$\theta=+1$.263	$\theta=+1$.263 0	$\theta=+1$.397	$\theta=+1$.397 0
		Item4=1	Item4=0			Item4=1	Item4=0			Item4=1	Item4=0
$\theta=-1$.019	.169	$\theta=-1$.048	0	$\theta=-1$.010	0
$\theta=0$.267	.400	$\theta=0$.688	0	$\theta=0$.592	0
$\theta=+1$.102	.044	$\theta=+1$.263	0	$\theta=+1$.397	0
		Item5=1	Item5=0			Item5=1	Item5=0			Item5=1	Item5=0
$\theta=-1$.019	.169	$\theta=-1$.005	.044	$\theta=-1$.010	0
$\theta=0$.267	.400	$\theta=0$.275	.413	$\theta=0$.592	0
$\theta=+1$.102	.044	$\theta=+1$.184	.079	$\theta=+1$.397	0

Table 4

Trace of Potential Tables and Clique Intersection Table for Five Responses to IED Items:
Response Vector = (0,0,1,1,1), Information from Three Most-Recent Items Only

INITIAL STATUS				AFTER ITEM 1 = 0				AFTER (0,0)			
		Item1=1	Item1=0			Item1=1	Item1=0			Item1=1	Item1=0
$\theta = -1$.025	.225	$\theta = -1$		0	.375	$\theta = -1$		0	.500
$\theta = 0$.200	.300	$\theta = 0$		0	.500	$\theta = 0$		0	.444
$\theta = +1$.175	.075	$\theta = +1$		0	.125	$\theta = +1$		0	.056
		Item2=1	Item2=0			Item2=1	Item2=0			Item2=1	Item2=0
$\theta = -1$.025	.225	$\theta = -1$.038	.337	$\theta = -1$		0	.500
$\theta = 0$.200	.300	$\theta = 0$.200	.300	$\theta = 0$		0	.444
$\theta = +1$.175	.075	$\theta = +1$.088	.038	$\theta = +1$		0	.056
		Item3=1	Item3=0			Item3=1	Item3=0			Item3=1	Item3=0
θ				θ				θ			
$\theta = -1$.250	$\theta = -1$.025 .225	$\theta = -1$.375	$\theta = -1$.038 .337	$\theta = -1$.500	$\theta = -1$.050 .450
$\theta = 0$.500	$\theta = 0$.200 .300	$\theta = 0$.500	$\theta = 0$.200 .300	$\theta = 0$.444	$\theta = 0$.178 .267
$\theta = +1$.250	$\theta = +1$.175 .075	$\theta = +1$.125	$\theta = +1$.088 .038	$\theta = +1$.056	$\theta = +1$.039 .017
		Item4=1	Item4=0			Item4=1	Item4=0			Item4=1	Item4=0
$\theta = -1$.025	.225	$\theta = -1$.038	.337	$\theta = -1$.050	.450
$\theta = 0$.200	.300	$\theta = 0$.200	.300	$\theta = 0$.178	.267
$\theta = +1$.175	.075	$\theta = +1$.088	.038	$\theta = +1$.039	.017
		Item5=1	Item5=0			Item5=1	Item5=0			Item5=1	Item5=0
$\theta = -1$.025	.225	$\theta = -1$.038	.337	$\theta = -1$.050	.450
$\theta = 0$.200	.300	$\theta = 0$.200	.300	$\theta = 0$.178	.267
$\theta = +1$.175	.075	$\theta = +1$.088	.038	$\theta = +1$.039	.017
AFTER (0,0,1)				AFTER (-,0,1,1)				AFTER (-,-,1,1,1)			
		Item1=1	Item1=0			Item1=1	Item1=0			Item1=1	Item1=0
$\theta = -1$		0	.188	$\theta = -1$.003	.023	$\theta = -1$.000	.002
$\theta = 0$		0	.667	$\theta = 0$.221	.331	$\theta = 0$.108	.163
$\theta = +1$		0	.146	$\theta = +1$.295	.127	$\theta = +1$.509	.218
		Item2=1	Item2=0			Item2=1	Item2=0			Item2=1	Item2=0
$\theta = -1$		0	.188	$\theta = -1$		0	.026	$\theta = -1$.000	.002
$\theta = 0$		0	.667	$\theta = 0$		0	.552	$\theta = 0$.108	.163
$\theta = +1$		0	.146	$\theta = +1$		0	.422	$\theta = +1$.509	.218
		Item3=1	Item3=0			Item3=1	Item3=0			Item3=1	Item3=0
θ				θ				θ			
$\theta = -1$.188	$\theta = -1$.188 0	$\theta = -1$.026	$\theta = -1$.026 0	$\theta = -1$.002	$\theta = -1$.002 0
$\theta = 0$.667	$\theta = 0$.667 0	$\theta = 0$.552	$\theta = 0$.552 0	$\theta = 0$.271	$\theta = 0$.271 0
$\theta = +1$.146	$\theta = +1$.146 0	$\theta = +1$.422	$\theta = +1$.422 0	$\theta = +1$.727	$\theta = +1$.727 0
		Item4=1	Item4=0			Item4=1	Item4=0			Item4=1	Item4=0
$\theta = -1$.019	.169	$\theta = -1$.026	0	$\theta = -1$.002	0
$\theta = 0$.267	.400	$\theta = 0$.552	0	$\theta = 0$.271	0
$\theta = +1$.102	.044	$\theta = +1$.422	0	$\theta = +1$.727	0
		Item5=1	Item5=0			Item5=1	Item5=0			Item5=1	Item5=0
$\theta = -1$.019	.169	$\theta = -1$.003	.023	$\theta = -1$.002	0
$\theta = 0$.267	.400	$\theta = 0$.221	.331	$\theta = 0$.271	0
$\theta = +1$.102	.044	$\theta = +1$.295	.127	$\theta = +1$.727	0

Table 5
Posterior Means for θ

t	0	1	2	3	4	5
No Decay	.00	-.25	-.44	-.04	.22	.39
Three most recent x 's only	.00	-.25	-.44	-.04	.40	.73
Geometric Decay, $\delta=.75$.00	-.25	-.42	.07	.35	.51
Attenuated posterior, $\delta=.75$.00	-.25	-.40	.13	.39	.53

Table 6
Conditional Probabilities Implied by Decaying Likelihood with $\delta=.75$

θ	Lag=0		Lag=1		Lag=2		Lag=3		Lag=4	
	x=1	x=0	x=1	x=0	x=1	x=0	x=1	x=0	x=1	x=0
-1	.100	.900	.161	.839	.226	.774	.284	.716	.331	.669
0	.400	.600	.425	.575	.444	.556	.458	.542	.468	.532
+1	.700	.300	.654	.346	.616	.384	.588	.412	.567	.433

Table 7

Trace of Potential Tables and Clique Intersection Table for Five Responses:
Response Vector = (0,0,1,1,1), Likelihood Decay with $\delta=.75$

INITIAL STATUS			AFTER ITEM 1 = 0			BEFORE ITEM 2		
		Item1=1 Item1=0			Item1=1 Item1=0			Item1=1 Item1=0
	$\theta=-1$.025 .225		$\theta=-1$	0 .375		$\theta=-1$.061 .314
	$\theta=0$.200 .300		$\theta=0$	0 .500		$\theta=0$.212 .288
	$\theta=+1$.175 .075		$\theta=+1$	0 .125		$\theta=+1$.082 .043
		Item2=1 Item2=0			Item2=1 Item2=0			Item2=1 Item2=0
	$\theta=-1$.025 .225		$\theta=-1$.038 .337		$\theta=-1$.038 .337
	$\theta=0$.200 .300		$\theta=0$.200 .300		$\theta=0$.200 .300
	$\theta=+1$.175 .075		$\theta=+1$.088 .038		$\theta=+1$.088 .038
		Item3=1 Item3=0			Item3=1 Item3=0			Item3=1 Item3=0
θ			θ			θ		
$\theta=-1$.250	$\theta=-1$.025 .225	$\theta=-1$.375	$\theta=-1$.038 .337	$\theta=-1$.375	$\theta=-1$.038 .337
$\theta=0$.500	$\theta=0$.200 .300	$\theta=0$.500	$\theta=0$.200 .300	$\theta=0$.500	$\theta=0$.200 .300
$\theta=+1$.250	$\theta=+1$.175 .075	$\theta=+1$.125	$\theta=+1$.088 .038	$\theta=+1$.125	$\theta=+1$.088 .038
		Item4=1 Item4=0			Item4=1 Item4=0			Item4=1 Item4=0
	$\theta=-1$.025 .225		$\theta=-1$.038 .337		$\theta=-1$.038 .337
	$\theta=0$.200 .300		$\theta=0$.200 .300		$\theta=0$.200 .300
	$\theta=+1$.175 .075		$\theta=+1$.088 .038		$\theta=+1$.088 .038
		Item5=1 Item5=0			Item5=1 Item5=0			Item5=1 Item5=0
	$\theta=-1$.025 .225		$\theta=-1$.038 .337		$\theta=-1$.038 .337
	$\theta=0$.200 .300		$\theta=0$.200 .300		$\theta=0$.200 .300
	$\theta=+1$.175 .075		$\theta=+1$.088 .038		$\theta=+1$.088 .038
AFTER (0,0)			BEFORE ITEM 3			AFTER (0,0,1)		
		Item1=1 Item1=0			Item1=1 Item1=0			Item1=1 Item1=0
	$\theta=-1$	0 .487			Item1=1 Item1=0		$\theta=-1$	0 .157
	$\theta=0$	0 .446			Item1=1 Item1=0		$\theta=0$	0 .618
	$\theta=+1$	0 .067			Item1=1 Item1=0		$\theta=+1$	0 .225
		Item2=1 Item2=0			Item2=1 Item2=0			Item2=1 Item2=0
	$\theta=-1$	0 .487			Item2=1 Item2=0		$\theta=-1$	0 .157
	$\theta=0$	0 .446			Item2=1 Item2=0		$\theta=0$	0 .618
	$\theta=+1$	0 .067			Item2=1 Item2=0		$\theta=+1$	0 .225
		Item3=1 Item3=0			Item3=1 Item3=0			Item3=1 Item3=0
θ			θ			θ		
$\theta=-1$.487	$\theta=-1$.049 .438	$\theta=-1$.487	$\theta=-1$.049 .438	$\theta=-1$.157	$\theta=-1$.157 0
$\theta=0$.446	$\theta=0$.178 .268	$\theta=0$.446	$\theta=0$.178 .268	$\theta=0$.618	$\theta=0$.618 0
$\theta=+1$.067	$\theta=+1$.047 .020	$\theta=+1$.067	$\theta=+1$.047 .020	$\theta=+1$.225	$\theta=+1$.225 0
		Item4=1 Item4=0			Item4=1 Item4=0			Item4=1 Item4=0
	$\theta=-1$.049 .438		$\theta=-1$.049 .438		$\theta=-1$.016 .141
	$\theta=0$.178 .268		$\theta=0$.178 .268		$\theta=0$.247 .371
	$\theta=+1$.047 .020		$\theta=+1$.047 .020		$\theta=+1$.158 .068
		Item5=1 Item5=0			Item5=1 Item5=0			Item5=1 Item5=0
	$\theta=-1$.049 .438		$\theta=-1$.049 .438		$\theta=-1$.016 .141
	$\theta=0$.178 .268		$\theta=0$.178 .268		$\theta=0$.247 .371
	$\theta=+1$.047 .020		$\theta=+1$.047 .020		$\theta=+1$.158 .068

(continued)

Table 7, continued

BEFORE ITEM 4

		lag=3	itm1=1	itm1=0
θ	$\theta=-1$.045	.112
	$\theta=0$.283	.335
	$\theta=+1$.132	.093
		lag=2	itm2=1	itm2=0
θ	$\theta=-1$.035	.122
	$\theta=0$.274	.344
	$\theta=+1$.139	.086
		lag=1	itm3=1	itm3=0
θ	$\theta=-1$.025	.132
	$\theta=0$.262	.356
	$\theta=+1$.147	.078
			itm4=1	itm4=0
θ	$\theta=-1$.016	.141
	$\theta=0$.247	.371
	$\theta=+1$.158	.068
			itm5=1	itm5=0
θ	$\theta=-1$.016	.141
	$\theta=0$.247	.371
	$\theta=+1$.158	.068

AFTER (0,0,1,1)

			itm1=1	itm1=0
θ	$\theta=-1$		0	.049
	$\theta=0$		0	.557
	$\theta=+1$		0	.394
			itm2=1	itm2=0
θ	$\theta=-1$		0	.049
	$\theta=0$		0	.557
	$\theta=+1$		0	.394
			itm3=1	itm3=0
θ	$\theta=-1$.049	0
	$\theta=0$.557	0
	$\theta=+1$.394	0
			itm4=1	itm4=0
θ	$\theta=-1$.049	0
	$\theta=0$.557	0
	$\theta=+1$.394	0
			itm5=1	itm5=0
θ	$\theta=-1$.005	.044
	$\theta=0$.223	.334
	$\theta=+1$.276	.118

BEFORE ITEM 5

		lag=4	itm1=1	itm1=0
θ	$\theta=-1$.016	.033
	$\theta=0$.261	.296
	$\theta=+1$.223	.171
		lag=3	itm2=1	itm2=0
θ	$\theta=-1$.014	.035
	$\theta=0$.255	.302
	$\theta=+1$.232	.162
		lag=2	itm3=1	itm3=0
θ	$\theta=-1$.011	.038
	$\theta=0$.247	.310
	$\theta=+1$.243	.151
		lag=1	itm4=1	itm4=0
θ	$\theta=-1$.008	.041
	$\theta=0$.237	.321
	$\theta=+1$.258	.136
			itm5=1	itm5=0
θ	$\theta=-1$.005	.044
	$\theta=0$.223	.334
	$\theta=+1$.276	.118

AFTER (0,0,1,1,1)

			itm1=1	itm1=0
θ	$\theta=-1$		0	.018
	$\theta=0$		0	.455
	$\theta=+1$		0	.526
			itm2=1	itm2=0
θ	$\theta=-1$		0	.018
	$\theta=0$		0	.455
	$\theta=+1$		0	.526
			itm3=1	itm3=0
θ	$\theta=-1$.018	0
	$\theta=0$.455	0
	$\theta=+1$.526	0
			itm4=1	itm4=0
θ	$\theta=-1$.018	0
	$\theta=0$.455	0
	$\theta=+1$.526	0
			itm5=1	itm5=0
θ	$\theta=-1$.018	0
	$\theta=0$.455	0
	$\theta=+1$.526	0

Table 8

Trace of Potential Tables and Clique Intersection Table for Five IID Responses:
Response Vector = (0,0,1,1,1), Attenuated Posterior Decay, $\delta=.75$

INITIAL STATUS				AFTER ITEM 1 = 0				BEFORE ITEM 2			

Table 8, continued

BEFORE ITEM 4

		Item1=1	Item1=0
$\theta=-1$.163	0	.163
$\theta=0$.578	0	.578
$\theta=+1$.259	0	.259
		Item2=1	Item2=0
$\theta=-1$.163	0	.163
$\theta=0$.578	0	.578
$\theta=+1$.259	0	.259
		Item3=1	Item3=0
θ			
$\theta=-1$.163	.163	0
$\theta=0$.578	.578	0
$\theta=+1$.259	.259	0
		Item4=1	Item4=0
$\theta=-1$.016	.016	.147
$\theta=0$.231	.231	.347
$\theta=+1$.181	.181	.078
		Item5=1	Item5=0
$\theta=-1$.016	.016	.147
$\theta=0$.231	.231	.347
$\theta=+1$.181	.181	.078

AFTER (0,0,1,1)

		Item1=1	Item1=0
$\theta=-1$.038	0	.038
$\theta=0$.539	0	.539
$\theta=+1$.423	0	.423
		Item2=1	Item2=0
$\theta=-1$.038	0	.038
$\theta=0$.539	0	.539
$\theta=+1$.423	0	.423
		Item3=1	Item3=0
θ			
$\theta=-1$.038	.038	0
$\theta=0$.539	.539	0
$\theta=+1$.423	.423	0
		Item4=1	Item4=0
$\theta=-1$.038	.038	0
$\theta=0$.539	.539	0
$\theta=+1$.423	.423	0
		Item5=1	Item5=0
$\theta=-1$.004	.004	.034
$\theta=0$.216	.216	.323
$\theta=+1$.296	.296	.127

BEFORE ITEM 5

		Item1=1	Item1=0
$\theta=-1$.091	0	.091
$\theta=0$.529	0	.529
$\theta=+1$.380	0	.380
		Item2=1	Item2=0
$\theta=-1$.091	0	.091
$\theta=0$.529	0	.529
$\theta=+1$.380	0	.380
		Item3=1	Item3=0
θ			
$\theta=-1$.091	.091	0
$\theta=0$.529	.529	0
$\theta=+1$.380	.380	0
		Item4=1	Item4=0
$\theta=-1$.091	.091	0
$\theta=0$.529	.529	0
$\theta=+1$.380	.380	0
		Item5=1	Item5=0
$\theta=-1$.009	.009	.082
$\theta=0$.212	.212	.318
$\theta=+1$.266	.266	.114

AFTER (0,0,1,1,1)

		Item1=1	Item1=0
$\theta=-1$.019	0	.019
$\theta=0$.435	0	.435
$\theta=+1$.546	0	.546
		Item2=1	Item2=0
$\theta=-1$.019	0	.019
$\theta=0$.435	0	.435
$\theta=+1$.546	0	.546
		Item3=1	Item3=0
θ			
$\theta=-1$.019	.019	0
$\theta=0$.435	.435	0
$\theta=+1$.546	.546	0
		Item4=1	Item4=0
$\theta=-1$.019	.019	0
$\theta=0$.435	.435	0
$\theta=+1$.546	.546	0
		Item5=1	Item5=0
$\theta=-1$.019	.019	0
$\theta=0$.435	.435	0
$\theta=+1$.546	.546	0

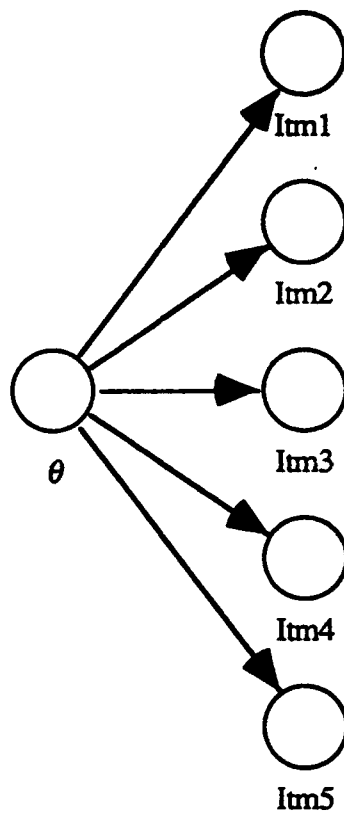


Figure 1

Directed Graph for the Educational Testing Example

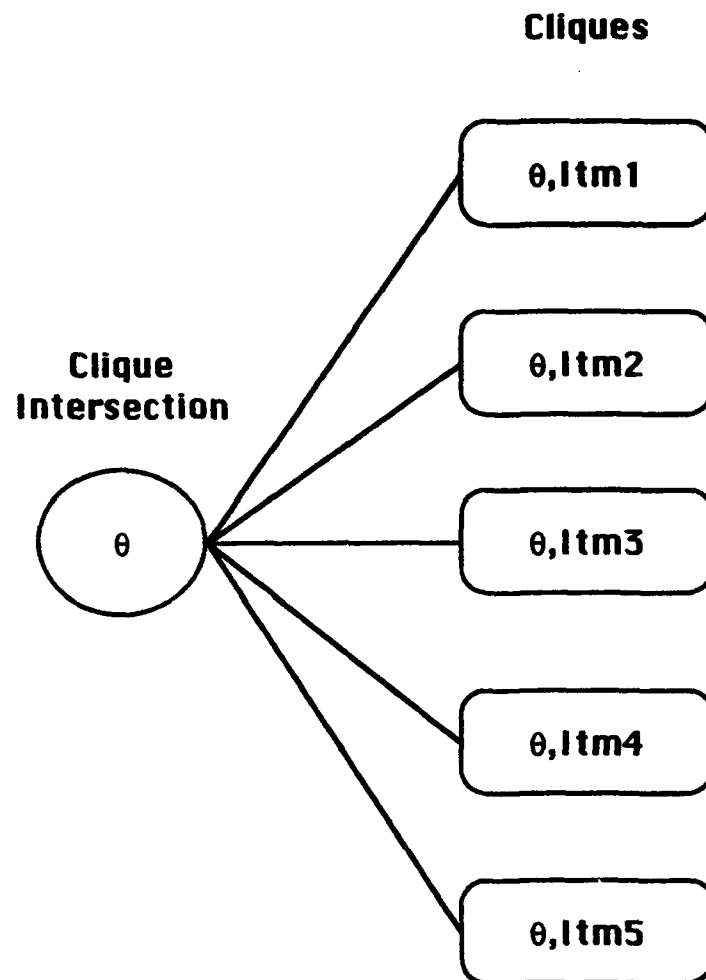


Figure 2

Join Tree for the Educational Testing Example

Brophy 05 April 94

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